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Iterative Method for Solving Parabolic Linear-Quadratic Optimal Control Problem with Constraints on the Time Derivative of the State

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Abstract—We consider a linear-quadratic optimal control problem of a system governed by parabolic equation with distributed in right-hand side control and control in Neumann boundary condition. Pointwise constraints for control functions and for time derivative of the state function are imposed. We construct a mesh approximation of this problem using two different approximations of the objective functional. Iterative solution methods are investigated for the constructed approximations of the optimal control problems. Numerical results confirm the effectiveness of the proposed methods.

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1. INTRODUCTION

Optimal control of time-dependent production processes plays an important role in many industrial applications such as continuous casting problem, crystal growth, cooling of glass melts etc. (see [1–5]). These processes are described by parabolic partial differential equations involving the temperature as a state variable. A need to avoid the defects of the product leads to pointwise constraints on the temperature variable. As a result, mathematical models of the processes appear as state constrained parabolic optimal control problems. Theoretical aspects of parabolic optimal control problems with pointwise constraints for state function are studied in [6–11] (see also the bibliography therein). Numerical solution methods are analysed in [12–20]. In particular, in the articles [12–15] error estimates are derived for discretizations of several classes of these problem, in [16, 17] convergence of regularization methods are proved. Uzawa-type iterative solution methods for finite dimensional problems approximating state constrained parabolic optimal control problems with pointwise constraints for state function are developed in the articles [18–20]. The parabolic optimal control problems with pointwise constraints for time derivative of the state function are considered in [21–23]. In these papers, convergence of Uzawa-type iterative solution methods for finite dimensional approximations of the mentioned problems are proved.

In this paper we consider a parabolic optimal control problem with distributed and boundary control and with observation in the domain. Constraints on the control and on time derivative of state are imposed. We approximate this problem by backward Euler finite difference scheme, prove the existence of a solution and develop iterative solution methods. We construct preconditioned Uzawa-type iterative

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